Interplay of the Rashba and Dresselhaus spin-orbit coupling in the optical spin susceptibility of 2D electron systems

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We present calculations of the frequency-dependent spin susceptibility tensor of a two-dimensional electron gas with competing Rashba and Dresselhaus spin-orbit interaction. It is shown that the interplay between both types of spin-orbit coupling gives rise to an anisotropic spectral behavior of the spin density response function which is significantly different from that of vanishing Rashba or Dresselhaus case. Strong resonances are developed in the spin susceptibility as a consequence of the angular anisotropy of the energy spin-splitting. This characteristic optical modulable response may be useful to experimentally probe spin accumulation and spin density currents in such systems.

Electrical manipulation of the electron and hole spins without the need of ferromagnetic materials and/or external magnetic fields is nowadays one of the central aspects in the field of spintronics. [1, 2, 3] The presence of a sizeable spin-orbit interaction (SOI) in low-dimensional semiconductor structures and its modulation possibility (through electrical gating) make it a very prominent mechanism for the access and manipulation of the carriers spin states.

It has been established that the dominant contributions to the SOI in quasi- two dimensional electron gases (2DEG) are the so called Rashba and Dresselhaus SO couplings.[4] The former results from the asymmetry of the confining potential that creates the 2DEG, while the latter arises due to the inversion asymmetry of the bulk. Several interesting effects and spin-based devices relying in these SOI mechanisms have been predicted and proposed in the last few years. For instance, the celebrated spin transistor proposed by Datta and Das [5], and its recent non-ballistic version [6]. An intrinsic spin Hall effect in which a transverse spin current is driven by a dc electric field (without a net charge current) has been also predicted to occur due to SOI effects. [7, 8, 9] More recently, a spin (Hall) accumulation has been observed through optical measurements[10, 11, 12], and lately, a purely electrical detection of a spin Hall current has been reported. [13] Electric-field-induced spin orientation in SOI coupled systems [14, 15, 16, 17, 18] and strained semiconductors has been also explored. [19]

On the other hand, the spin-splitting caused by SOI in electron systems opens the possibility of resonant effects via transitions between the spin-split states as a response to alternating electric fields. [20, 21, 22, 23, 24, 25] The importance of the study of such SOI effects in the dynamical regime (frequency dependent response) has been emphasized by several authors studying a variety of physical aspects, including spin and charge optical conductivities [22], optical absorption spectra [25, 26], optical control of the spin Hall current through intense ac probing fields [27], electron-electron interaction effects [28, 29], electron-phonon interaction on spin Hall currents [30], plasmon modes [23, 31, 32, 33], and the relation between the spin Hall conductivity and the spin susceptibility [28, 29, 34] or the dielectric function [24].

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The spin susceptibility plays a central role of the spin dynamics in a 2DEG. It gives the average spin polarization induced via electric-dipole or magnetic-dipole interactions. Thus, it can be used to obtain a magnetic susceptibility [28] or the electric-field-induced spin orientation factor.[16, 35] Moreover, other transport properties like charge or spin Hall conductivities can also be expressed in terms of such spin response function.[34]

Following S. I. Erlingsson et al.[34], in this paper we report on the analytical and numerical calculations of the frequency-dependent spin susceptibility tensor of a 2DEG with Rashba and Dresselhaus SOI. In Ref. [34] expressions for the tensor components were obtained, however only approximated results were reported in the finite frequency regime. Their analytical expressions for the spin susceptibilities are valid as long as $k_{SO}/k_F << 1$ and $\alpha << \beta$, where k_{SO} and k_F are the characteristic spin-orbit coupling and Fermi wave numbers, while α and β are the Rashba and Dresselhaus SOI strength parameters, respectively.

Here we show that in the more general case, particularly when there is a strong interplay between the Rashba and Dresselhaus SOI, very distinctive features of the optical spin susceptibility spectra arises in the system. This suggests that an optically modulable spin density response may be achievable in such systems. Furthermore, the calculated spectra show that the combination of the excitation at finite frequency and the interplay between the Rashba and Dresselhaus couplings could also be used for measuring the ratio between the SO coupling parameters.

We consider a 2D free electron system lying at the $z = 0$ plane subjected to spin-orbit interaction, with a Hamiltonian given by

$$H = \frac{\hbar^2(k_x^2 + k_y^2)}{2m^*}\,\mathbf{I} + \alpha(k_x\sigma_y - k_y\sigma_x) + \beta(k_x\sigma_x - k_y\sigma_y) \quad , \tag{1}$$

where k_x, k_y are the components of the 2D electron wave vector, $\mathbf{I}$ is the 2×2 unit matrix and σ_μ are the Pauli matrices. The second term corresponds to the Rashba SO coupling which originates from any source of structural inversion asymmetry (SIA) of the confining potential. The third term is the linear Dresselhaus coupling which results from bulk-induced inversion asymmetry (BIA) in a narrow [001] quantum well. Spin-orbit interaction appears as a relativistic correction (derived from the Dirac equation) to the Hamiltonian of a slow electron. It includes the gradient of a potential in which the electron moves. In atomic physics such term leads to the well known $\mathbf{L}\cdot\mathbf{S}$ coupling between the orbital and intrinsic angular momentum due to the Coulomb potential. For an electron in a crystal environment there are several sources of potential gradient (impurities, confinement, boundaries, external electric field) which lead to an enhancement of SO coupling in solids. For quasi-2D systems the more significant contributions are those due to SIA (Rashba) and BIA (Dresselhaus).[4]

The eigenstates $|\mathbf{k}\lambda\rangle$ for the in-plane motion are specified by the wave vector $\mathbf{k} = (k_x, k_y) = k(\cos\theta, \sin\theta)$ and chirality $\lambda = \pm$ of the spin branches. The double sign corresponds to the upper (+) and lower ($-$) parts of the energy spectrum given by

$$\varepsilon_\lambda(k,\theta) = \frac{\hbar^2}{2m^*}(k + \lambda k_{so}(\theta))^2 - \frac{\hbar^2 k_{so}^2(\theta)}{2m^*} \tag{2}$$

where $k_{so}(\theta) = m^*\Delta(\theta)/\hbar^2$ is the characteristic SO momentum, $\Delta^2(\theta) = \alpha^2 + \beta^2 - 2\alpha\beta\sin 2\theta$ describes the angular anisotropy of the spin splitting. At zero temperature, the two spin-split subbands are filled up to the same (positive) Fermi energy level ε_F but with different Fermi wave vectors $q_\lambda(\theta) = \sqrt{2m^*\varepsilon_F/\hbar^2 + k_{so}^2(\theta)} - \lambda k_{so}(\theta)$, determined from the equations $\varepsilon_\lambda(q_\lambda(\theta),\theta) = \varepsilon_F$. Here, $\varepsilon_F = \hbar^2(k_0^2 - 2q_{so}^2)/2m^*$ with $k_0 = \sqrt{2\pi n}$ being the Fermi wave vector of a spin-degenerate 2DEG with electron density n, and $q_{so} = m^*\sqrt{\alpha^2+\beta^2}/\hbar^2$. The SOI splits the Fermi line into two curves with radii given by $q_\lambda(\theta)$ which, as the energy surfaces $\varepsilon_\lambda(\mathbf{k})$, are symmetric with respect to the (1,1) and (-1,1) directions in $\mathbf{k}-$space (Fig. 1). When α or β is null, the dispersions are isotropic and the Fermi contours

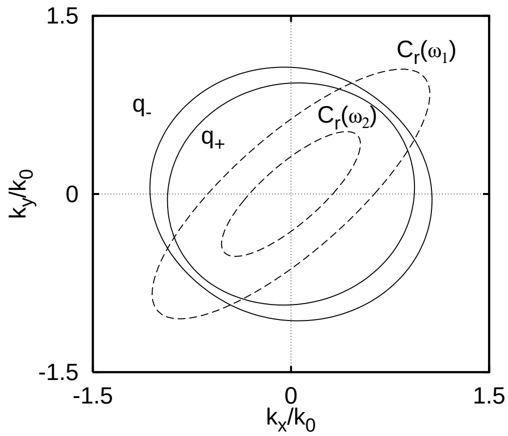

Fig. 1 Fermi contours $q_\lambda(\theta)$ and the constant-energy-difference curve $C_r(\omega)$ defined by $\varepsilon_+(\mathbf{k}) - \varepsilon_-(\mathbf{k}) = \hbar\omega$, shown for two values of the photon energy, $\hbar\omega_1 > \hbar\omega_2$. $C_r(\omega)$ is a rotated ellipse with semi-axis of lengths (major) $k_a(\omega) = \hbar\omega/2|\alpha-\beta|$ and (minor) $k_b(\omega) = \hbar\omega/2|\alpha+\beta|$ oriented along the $(1,1)$ and $(-1,1)$ directions respectively. The sample parameters used here are $n = 5\times10^{11}\text{cm}^{-2}$, $\alpha = 1.6\times10^{-9}$ eV cm, $\beta = 0.5\alpha$ and $m^* = 0.055m$.

are concentric circles. If $\alpha = \pm\beta$ the spin-splitting along the ($\pm$1,1) direction vanishes and the spin-split dispersion branches are two circles with the same radius and displaced from the origin (along ($\mp$1,1) direction).

Within the linear response Kubo formalism the spin susceptibility is given by

$$\chi_{\mu\mu'}(\omega) = \frac{i}{\hbar}\int_0^\infty dt\, e^{i(\omega+i\eta)t}\langle[\sigma_\mu(t),\sigma_{\mu'}(0)]\rangle \quad , \qquad \mu,\mu' = x,y \tag{3}$$

where the symbol $\langle\cdots\rangle = \Sigma_\lambda\int d^2k\, f(\epsilon_\lambda(k))(\cdots)$ indicates quantum and thermal averaging, $f(\epsilon)$ is the Fermi distribution function, and $\eta\to0^+$. This is a spin-spin response function for a spatially homogeneous (in-plane) perturbation oscillating at frequency ω.

In the limit of vanishing temperature, eq. (3) takes the form

$$\chi_{\mu\mu'}(\omega) = \frac{1}{\pi^2}\int' d^2k\,\langle-|\sigma_\mu|+\rangle\langle+|\sigma_{\mu'}|-\rangle\,\frac{\varepsilon_+(\mathbf{k})-\varepsilon_-(\mathbf{k})}{[\varepsilon_+(\mathbf{k})-\varepsilon_-(\mathbf{k})]^2-[\hbar(\omega+i\eta)]^2} \quad , \tag{4}$$

the prime on the integral indicates that integration is restricted to the region between the Fermi contours, $q_+(\theta) < k < q_-(\theta)$, for which $\varepsilon_-(\mathbf{k}) < \varepsilon_F < \varepsilon_+(\mathbf{k})$, (Fig. 1).

Using the result

$$\langle-|\sigma_\mu|+\rangle = -\langle+|\sigma_\mu|-\rangle = \frac{i}{\Delta(\theta)}[\delta_{\mu x}(\alpha\cos\theta-\beta\sin\theta)+\delta_{\mu y}(\alpha\sin\theta-\beta\cos\theta)] \tag{5}$$

the susceptibility tensor becomes

$$\chi_{\mu\mu'}(\omega) = \frac{1}{\pi^2}\int_0^{2\pi}d\theta\,\frac{g_{\mu\mu'}(\theta)}{\Delta(\theta)}\int_{q_+(\theta)}^{q_-(\theta)}dk\,\frac{k^2}{4k^2\Delta^2(\theta)-[\hbar(\omega+i\eta)]^2} \quad , \tag{6}$$

where

$$\begin{aligned} g_{\mu\mu'}(\theta) \quad &= \quad \delta_{\mu\mu'}[\delta_{\mu x}(\alpha\cos\theta-\beta\sin\theta)^2+\delta_{\mu y}(\alpha\sin\theta-\beta\cos\theta)^2] \\ &\qquad +(1-\delta_{\mu\mu'})(\alpha\cos\theta-\beta\sin\theta)(\alpha\sin\theta-\beta\cos\theta) \quad . \end{aligned}$$

It can be shown that $\chi_{xx}(\omega) = \chi_{yy}(\omega)$ and $\chi_{xy}(\omega) = \chi_{yx}(\omega)$. Note also that for $\beta = 0$, $\chi_{xy}(\omega) = \chi_{yx}(\omega) = 0$.

We can write the susceptibility in the form $\chi_{\mu\mu'} = \chi'_{\mu\mu'} + i\chi''_{\mu\mu'}$. The real part is

$$\chi'_{\mu\mu'}(\omega) = \chi_{\mu\mu'}(0) + \frac{\hbar\omega}{16\pi^2}\int_0^{2\pi} d\theta\, \frac{g_{\mu\mu'}(\theta)}{\Delta^4(\theta)}\, \log\left|\frac{[\omega+\Omega_+(\theta)][\omega-\Omega_-(\theta)]}{[\omega+\Omega_-(\theta)][\omega-\Omega_+(\theta)]}\right| \tag{7}$$

where $\hbar\Omega_\pm = |\varepsilon_F - \varepsilon_\mp(q_\pm(\theta),\theta)| = 2q_\pm(\theta)\Delta(\theta)$ and the static value is

$$\chi_{\mu\mu'}(0) = \frac{\nu_0}{2}\left[\delta_{\mu\mu'} - (1-\delta_{\mu\mu'})\left(\frac{\beta}{\alpha}\Theta(\alpha^2-\beta^2) + \frac{\alpha}{\beta}\Theta(\beta^2-\alpha^2)\right)\right] \quad , \tag{8}$$

$\nu_0 = m^*/\pi\hbar^2$ is the density of states of a spin-degenerate 2DEG, and $\Theta(x)$ is the unit step function, $\Theta(x) = 1$ if $x > 0$ and $\Theta(x) = 0$ if $x < 0$.

For the imaginary part we have

$$\chi''_{\mu\mu'}(\omega) \;=\; \pi \int' \frac{d^2k}{(2\pi)^2}\, \langle -|\sigma_\mu|+\rangle\langle +|\sigma_{\mu'}|-\rangle\, \delta(\varepsilon_+(\mathbf{k}) - \varepsilon_-(\mathbf{k}) - \hbar\omega) \tag{9}$$

$$\;=\; \frac{\hbar\omega}{16\pi}\int d\theta\, \frac{g_{\mu\mu'}(\theta)}{\Delta^4(\theta)}\, \Theta[\hbar\omega - \hbar\Omega_+(\theta)]\, \Theta[\hbar\Omega_-(\theta) - \hbar\omega] \quad . \tag{10}$$

These equations express the fact that the only transitions allowed between spin-split subbands ε_λ due to photon absorption at energy $\hbar\omega$ are those for which $\hbar\Omega_+(\theta) \le \hbar\omega \le \hbar\Omega_-(\theta)$. That is, for a given ω only those angles satisfying this condition must be considered in the integral (10), see Fig. 2c. This is different to the pure Rashba (or Dresselhaus) case, where the whole interval $[0, 2\pi]$ contributes to the integral for each allowed photon energy. The non-isotropic spin-splitting originated by the simultaneous presence of both coupling strengths, forces the optical excitation to be $\mathbf{k}-$selective.

In Fig. 2 we show $\chi_{xx}(\omega)$ as obtained from eqs. (7)-(10), the xy component behaves similarly. The result is remarkably different from that of the pure Rashba or Dresselhaus case, where the spin-splitting is isotropic in the momentum space. For example, if $\beta = 0$, $\alpha \ne 0$, then $\chi''_{\mu\mu'}(\omega) = \chi_R\delta_{\mu\mu'}$ only for $2\alpha k_+ \le \hbar\omega \le 2\alpha k_-$, otherwise it vanishes, where $\chi_R = \hbar\omega/16\alpha^2$, with $k_\pm = q_\pm(\beta = 0)$ being independent of angle θ; (see Fig. 3). Thus, in this case the width $\Delta\mathcal{E}$ of the spectrum is $\Delta\mathcal{E}_R = 4\varepsilon_R$ (or $4\varepsilon_D$ if $\alpha = 0$, $\beta \ne 0$); $\varepsilon_R = m^*\alpha^2/\hbar^2$ and $\varepsilon_D = m^*\beta^2/\hbar^2$ are the SO characteristic energy scales for the Rashba and Dresselhaus coupling. As was discussed in Ref. [22], $\Delta\mathcal{E}_{R,D}$ can be about an order of magnitude smaller than the width of the spectrum shown in Fig. 2b. Assuming that $\alpha > \beta$ and $(k_{so}(\theta)/k_0)^2 \ll 1$, we have $\Delta\mathcal{E} = 4\beta k_0 + \Delta\mathcal{E}_R + \Delta\mathcal{E}_D$ (if $\alpha < \beta$ the first term changes to $4\alpha k_0$). Thus, the absorption bandwidth could be manipulated by tuning the coupling strength α and/or through variations of the electron density $n = k_0^2/2\pi$. This fact could also be used to determine the sign of $\alpha - \beta$.[22]

To understand the structure of the spectra of Fig. 2, we further note that, according to eq. (9), the minimum (maximum) photon energy $\hbar\omega_+$ ($\hbar\omega_-$) required to induce optical transitions between the initial $\lambda = -$ and the final $\lambda = +$ subband corresponds to the excitation of an electron with wave vector lying on the q_+ (q_-) Fermi line at $\theta_+ = \pi/4$ or $5\pi/4$ ($\theta_- = 3\pi/4$ or $7\pi/4$), giving $\hbar\omega_\pm = \hbar\Omega_\pm(\theta_\pm) = 2k_0|\alpha\mp\beta| \mp 2m^*(\alpha\mp\beta)^2/\hbar^2$. The absorption edges in the spectrum of Fig. 2b correspond exactly to $\hbar\omega_\pm$. The function $\chi''_{\mu\mu'}(\omega)$ can also be written as a line integral along the arcs of the resonant curve $C_r(\omega)$ lying within the region enclosed by the Fermi lines $q_\lambda(\theta)$; see Fig. 1. The peaks observed in Fig.2b correspond to electronic excitations involving states with allowed wave vectors on $C_r(\omega)$ such that $|\nabla_{\mathbf{k}}(\varepsilon_+ - \varepsilon_-)|$ takes its minimum value. The first (second) peak is at a photon energy $\hbar\omega_a$ ($\hbar\omega_b$) for which the major (minor) semi-axis of the ellipse $C_r(\omega)$ (Fig. 1) coincides with the Fermi line $q_-(\theta_+)$ $(q_+(\theta_-))$, hence $\hbar\omega_a = \hbar\Omega_-(\theta_+) = 2k_0|\alpha-\beta| + 2m^*(\alpha-\beta)^2/\hbar^2$ and $\hbar\omega_b = \hbar\Omega_+(\theta_-) = 2k_0|\alpha+\beta| - 2m^*(\alpha+\beta)^2/\hbar^2$. The spectrum of $\chi''_{xx}(\omega)$ looks very similar to the joint density of states for the spin-split bands $\varepsilon_\pm$.[22] The unequal splitting at the Fermi level along the symmetry $(1,1)$ and $(-1,1)$ directions is thus responsible for

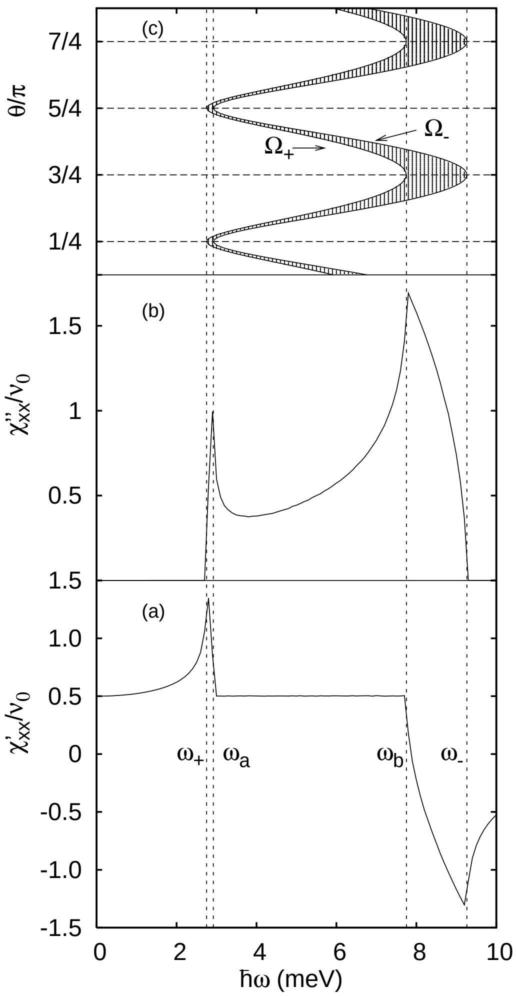

Fig. 2 (c) Angular region (shaded) in $\mathbf{k}-$space available for direct transitions as a function of photon energy. Only the shaded region contribute to the optical absorption [eq. (10)]. The energy boundaries are given by $\hbar\Omega_+(\theta) = \varepsilon_F - \varepsilon_-(q_+(\theta),\theta)$ and $\hbar\Omega_-(\theta) = \varepsilon_+(q_-(\theta),\theta) - \varepsilon_F$. (b) Imaginary and (a) Real part of the optical spin susceptibility $\chi_{xx}(\omega)$, $\nu_0 = m^*/\pi\hbar^2$. For the frequencies $\omega_+ = \Omega_+(\pi/4)$, $\omega_a = \Omega_-(\pi/4)$, $\omega_b = \Omega_+(3\pi/4)$, $\omega_- = \Omega_-(3\pi/4)$, see the text. The sample parameters are the same as in Fig. 1 .

the peaks at photon energies $\hbar\omega_a$ and $\hbar\omega_b$ respectively, giving meaning to the structure of the spectrum. The overall magnitude and the asymmetric shape of the spectrum are due to the factor $g_{\mu\mu'}(\theta)/\Delta^4(\theta)$ in eq.(10). The results for several values of β/α are shown in Fig. 3.

The real part of $\chi_{\mu\mu'}(\omega)$ presents additional spectral features. For photon energies in the range $\hbar\omega_a \leq \hbar\omega \leq \hbar\omega_b$ we find numerically that it takes the constant values $\chi'_{xx}(\omega) = \nu_0/2$ and $\chi'_{xy}(\omega) = -(\nu_0/2)(\alpha^2 + \beta^2)/2\alpha\beta$. The spectral characteristics of the response displayed in Fig. 2a shows that the magnitude and the direction of the dynamic spin magnetization could be modified via electrical gating and/or by adjusting the exciting frequency. This suggests new possibilities of electrical manipulation of the spin orientation in a 2DEG in the presence of competing Rashba and Dresselhaus SO couplings.

Following Ref. [36] we have also obtained the static value of $\chi_{\mu\mu'}(\omega)$ for finite momentum relaxation rate $\eta > 0$ (see eq. (6)). This parameter accounts phenomenologically for dissipation effects due to

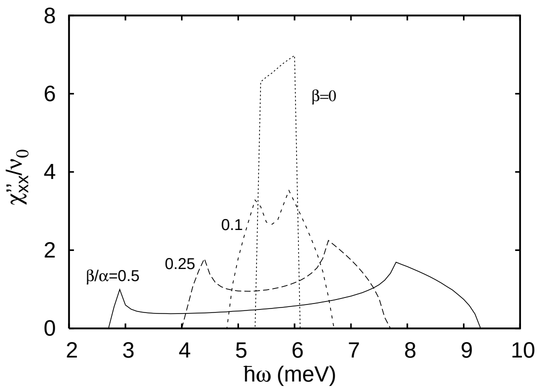

Fig. 3 Imaginary part of the spin susceptibility $\chi_{xx}(\omega)$ for several values of the ratio β/α. Other parameters are as those used in Fig. 1 .

impurity scattering. We found that, to linear order in $\varepsilon_{R,D}/\hbar\eta$, it vanishes as ($\alpha \neq \beta \neq 0$)

$$\chi_{xx}(0;\eta) \approx 4\nu_0 \left(\frac{\varepsilon_F}{\hbar\eta}\right)\left(\frac{\varepsilon_R+\varepsilon_D}{\hbar\eta}\right) \qquad \chi_{xy}(0;\eta) \approx -4\nu_0 \left(\frac{\varepsilon_F}{\hbar\eta}\right)\frac{2\sqrt{\varepsilon_R\varepsilon_D}}{\hbar\eta} \quad . \tag{11}$$

It is also possible to relate the spin current response to the spin density response. The definition of the spin conductivity $\sigma^{s,z}_{iy}(\omega)$ describing a $z-$polarized-spin current flowing in the $i-$direction as a response to the field $E(\omega)\hat{\mathbf{y}}$ involves the commutator $[\mathcal{J}^z_i(t), j_y(0)]$, where $j_i = ev_i$ and $\mathcal{J}^z_i = \hbar\{\sigma_z, v_i\}/4$ are the charge and spin current operators, respectively. Using the velocity operator $\mathbf{v}(\mathbf{k}) = \nabla_{\mathbf{k}}H/\hbar = \hbar\mathbf{k}/m^* + \hat{\mathbf{x}}(\beta\sigma_x + \alpha\sigma_y)/\hbar - \hat{\mathbf{y}}(\alpha\sigma_x + \beta\sigma_y)/\hbar$, this commutator can be written in terms of the correlators $[\sigma_i(t), \sigma_j(0)]$, ($i = x, y$), which determines the spin susceptibility (3). Thus, the following relations can be derived

$$\frac{\sigma^{s,z}_{xy}(\omega)}{e/8\pi} = \left(\frac{\alpha^2+\beta^2}{\alpha^2-\beta^2}\right)\frac{\chi_{xx}(\omega)}{\nu_0/2} + \left(\frac{2\alpha\beta}{\alpha^2-\beta^2}\right)\frac{\chi_{xy}(\omega)}{\nu_0/2} \tag{12}$$

$$\frac{\sigma^{s,z}_{yy}(\omega)}{e/8\pi} = \left(\frac{2\alpha\beta}{\alpha^2-\beta^2}\right)\frac{\chi_{xx}(\omega)}{\nu_0/2} + \left(\frac{\alpha^2+\beta^2}{\alpha^2-\beta^2}\right)\frac{\chi_{xy}(\omega)}{\nu_0/2} \quad . \tag{13}$$

These expressions are formally equivalent to eqs. (39) and (40) of Ref. [34]. This connection is very convenient because a spin polarization is more experimentally accesible than a spin current.

In summary, we have calculated the finite frequency spin susceptibility tensor of a two-dimensional electron gas with competing Rashba and Dresselhaus spin-orbit interaction. We find that the angular anisotropy of the energy spin-splitting introduced by the interplay between both SO coupling strengths yields a finite-frequency response with spectral features that are significantly different from that of a pure Rashba (Dresselhaus) coupling case. As a consequence, an optically modulable spin density response is then achievable in such systems which may be useful for spintronics applications.

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